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Original Paper

Characteristics of the Hydraulic Jump in Trapezoidal Channel Section

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Abstract

In this study, characteristics of the hydraulic jump in trapezoidal channel sections were analyzed and a general equation represents the solution of the hydraulic jump in the channels of arbitrary cross-sections (rectangular, triangular & trapezoidal) was driven depending on the momentum principle. The solution of the models was provided using Newton Raphson method. Consequently, Tables and charts of family curves of the conjugate depths ratio ($r=y_2/y_1$) have been prepared, for a very wide range values of Froude numbers and section ratios ($k=b/zy$). For each type of cross sections, the efficiency of the energy dissipation of the hydraulic jump was also analyzed and compared with each others. The relationship between the initial and sequent Froude numbers (F_{D1} and F_{D2}) has been indicated for various values of $k_1=b/zy_1$. Depending on the results of conjugate depths ratio $r = y_2 / y_1$, the length of the hydraulic jump were estimated for a very wide range of $k_1=b/zy_1$, using two suggested models. It was found that the channel shape has insignificant effect on the efficiency of the energy dissipation of the hydraulic jump, although the triangular section tends to be more efficient than the others by about 10 percent in higher F_{D1}. When ($F_{D1} > 6$), the velocity head after the jump could be neglected. When the section ratio k_1 is approximately 3, the length ratio of the hydraulic jump (L_j / y_2) reaches to a maximum value independent on the value of F_{D1}. In all cases, it was shown that the comparison of the theoretical results with other experimental data indicate a very good agreement

Key words: hydraulic jump - sequent depth ratio - jump in trapezoidal and triangular channels - Conjugate Depth, Energy Dissipaters

Introduction

The hydraulic jump is a natural phenomenon which may be defined as a sudden and turbulent passage of water from supercritical flow to subcritical state, (Modi, 2004). The abrupt change in flow condition is accompanied by considerable turbulence and energy losses. The hydraulic jump commonly occurs with natural flow conditions and with proper design can be an effective means of dissipating energy at hydraulic structures. Expressions for computing the before and after jump depth ratio (conjugate depths) and the length of jump are needed to design energy dissipaters that induce a hydraulic jump. For this reason, the hydraulic jump is often employed to dissipate energy and control erosion at storm water management structures.

Hydraulic jumps are commonly experienced in rivers, canals, industrial applications and manufacturing processes. (Montes, 1979; Chow, 1994; Treske, 1994; Reinaur and Hager, 1995; Chanson and Montes, 1995; Chanson, 2007 and Murzyn, 2007; studied the undular hydraulic jump, described its characteristics where the values of the Froude number in which the jump is no longer undular was calculated neglecting the effect of the channel width. The jump height, however, may be predicted quite accurately using momentum theory alone Hotchkiss *et al.,* (2003). Typically, the discharge and upstream depth are already known, and what remains to be determined is the downstream "sequent depth", Chadwick *et al.,* (2004).

The purpose of this study, is to develop a general solution of the sequent depth problem in trapezoidal channel section

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(rectangular, triangular & trapezoidal), based on the momentum principle law. Such a solution will be useful to analyze the characteristics flow of a turbulent hydraulic jump and to determine the length of the hydraulic jump as well as the efficiency dissipation.

Momentum Principle

Because of energy losses, the size and location of the hydraulic jump cannot be predicted using the energy equation. However, because momentum is conserved across hydraulic jumps under the assumptions of this study, momentum theory could be applied to determine the jump size and location Hotchkiss *et al.,* (2003). Figure 1 indicates the control volume used and the forces involved. Distribution of pressure in the upstream and downstream sections is assumed to be hydrostatic. So, applying the momentum equation in a frictionless channel considering the above assumptions, leads the momentum equation in the term of the specific force to be:

$$\frac{Q^2}{gA_1} + Z_{C1}A_1 = \frac{Q^2}{gA_2} + Z_{C2}A_2 = F \qquad (1)$$

$$\text{Or} \quad F_1 = F_2 \qquad (2)$$

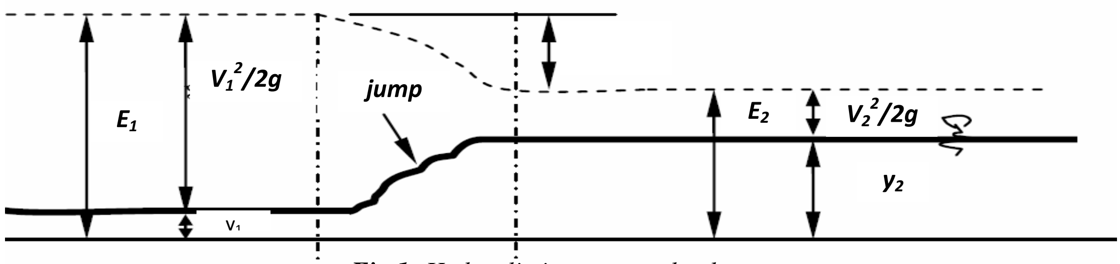

Fig.1: *Hydraulic jump control volume*

Where:

F: Specific force

Q: Flow rate

g: Gravity acceleration

A_1 & A_2: Cross-sectional area before and after the jump, respectively.

Z_{C1} & Z_{C2}: Distances of the centroids sections from the free surface area before and after the jump, respectively.

Consider that:

$$A = by + zy^2 \qquad (3)$$

$$T = b + 2zy \qquad (4)$$

$$F_r = \frac{V}{\sqrt{gy}} \qquad (5)$$

$$F_D = \frac{V}{\sqrt{gD}} \qquad (6)$$

Where:

T: Top width of the sectional area.

b: Bottom width of the sectional area.

z: side slope

V: Mean velocity.

F_r: Froude number in term of the depth of flow y.

F_D: Froude number in term of the hydraulic depth $D = A/T$.

Now, define a dimensionless factor ***k*** to be a section ratio such that:

$$k = \frac{b}{z\ y} \qquad (7)$$

Consequently, Eqs. (3 & 4) could be rewritten as:

$$A = zy^2(k+1) \qquad (8)$$

$$T = zy(k+2) \qquad (9)$$

Also, it could be seen that:

$$F_D = \sqrt{\frac{k+2}{k+1}}\ F_r \qquad (10)$$

According to the section ratio ***k***, the shape of the channel section will take the following form:

when $k = 0$, the section is a triangular shape.

and, when $k = \infty$, the section is a rectangular shape.

While for $0 < k < \infty$, the section is a trapezoidal shape.

By taking the moments about the top axis of a trapezoidal channel section, the centroid Position *Zc,* could be determined as:

$$Z_c = \frac{\left(\frac{1}{3} + \frac{1}{2}k\right)}{k+1}\ y \qquad (11)$$

Substituting the values of various terms of Eq. 2, considering Eqs. (7 to 11) and simplifying, the specific force before the jump F_1 will take the following form:

$$F_1 = Z\, y_1^{3}\left[\frac{F_r^{2}\left(k^2+3k+2\right)+\left(\frac{1}{2}k^2+\frac{4}{3}k+\frac{2}{3}\right)}{(k+2)}\right]_1 \quad (12)$$

By the same way, it could be seen that:

$$F_2 = Z\, y_2^{3}\left[\frac{F_r^{2}\left(k^2+3k+2\right)+\left(\frac{1}{2}k^2+\frac{4}{3}k+\frac{2}{3}\right)}{(k+2)}\right]_2 \quad (13)$$

Where the subscripts 1 & 2, refer to the corresponding variable of section 1 and 2 respectively. It is necessary now to represent the variables of Eq.13 in term of the same variables of the section 1, considering that:

$$k_2 = \frac{b}{zy_2} = r^{-1}k_1 \quad (14)$$

Where:

r: Conjugate depths ratio of the initial and sequent depths, (y_2 / y_1) .

Also, it could be seen that:

$$\frac{A_1^{\,2}}{A_2^{\,2}} = r^{-2}\left(\frac{k_1+1}{k_1+r}\right)^2 \quad (15)$$

$$F_{r_2}^{\;2} = r^{-1}\frac{A_1^{\,2}}{A_2^{\,2}}F_{r_1}^{\;2} \quad (16\text{ a})$$

or $$F_{r_2}^{\;2} = r^{-3}\left(\frac{k_1+1}{k_1+r}\right)^2 F_{r_1}^{\;2} \quad (16\text{ b})$$

So, Eq.13 will take the following form:

$$F_2 = Z\, y_2^{3}\left[\frac{r^{-2}F_r^{\;2}\left(\frac{k+1}{k+r}\right)^2\left(r^{-2}k^2+3r^{-1}k+2\right)+\left(\frac{1}{2}r^{-1}k^2+\frac{4}{3}k+\frac{2}{3}r\right)}{(k+2r)}\right]_1 \quad (17)$$

Satisfy the condition of Eq.2, taking in the count Eqs. (10, 12, & 17), the following equation is produced after some tedious mathematical steps:

$$r^4+\left(\frac{5k}{2}+1\right)r^3+\left(\frac{3k}{2}+1\right)(k+1)\,r^2+\left(\frac{k^2}{2}+\left(k-3F_D^{\;2}\frac{(k+1)}{(k+2)}\right)(k+1)\right)r-3F_D^{\;2}\frac{(k+1)^3}{(k+2)}=0 \quad (18)$$

Equation 18 represents the relationship of the Conjugate depths ratio of a hydraulic jump in a horizontal trapezoidal channel. This equation could be simplified by considering that:

$$B = \left(\frac{5k}{2}+1\right) \quad (19\text{ a})$$

$$C = \left(\frac{3k}{2}+1\right)(k+1) \quad (20\text{ a})$$

$$D = \left(\frac{k^2}{2}+\left(k-3F_D^{\;2}\frac{(k+1)}{(k+2)}\right)(k+1)\right) \quad (21\text{ a})$$

$$E = -3F_D^{\;2}\frac{(k+1)^3}{(k+2)} \quad (22\text{ a})$$

Where k is k_1 and F_D is F_{D1}. So, Eq. 18 will reduce to the following form:

$$r^4+B\,r^3+C\,r^2+D\,r+E=0 \quad (23\text{ a})$$

Conjugate Depths - Initial and Sequent Depths:

For a given values of F_{D1} and k_1, the solution of Eqs. (18 or 23a) represents the conjugate depths ratio $r = y_2/y_1$. As it is known, this Equation has four roots. The signs of the second and the third term of Eq.23a *(B & C)* are always positive, while the fifth term *E*, is always negative. The forth term *D*, may have a positive or a negative sign depending on the values of F_{D1} and k_1. According to Decard theory, equation 23 has always a unique positive root whatever the sign of the term D, and that is the required solution, (Hoffman, 2001). The researcher found that Newton–Raphson method is a very good technique to provide the results. Also, fixed-point method may be a useful alternative technique to determine the mathematical solution for the depths upstream and downstream of the hydraulic jump, (Vatankhah, 2008). Fig.2 represents a dimensionless chart for the conjugate depths ratio *r* for various upstream Froude numbers F_{D1}, corresponding to a very wide range of a section ratio k_1, from zero (i.e., triangular shape) to infinity (i.e., rectangular shape). As it is shown, the conjugate depths ratio has a little significant change at high section ratios for the same Froude numbers. Also, for all values of k_1, when $F_{D1} < 2$, the conjugate depths ratio *r* is near the corresponding value of the rectangular section. In case of the rectangular section *(where $k_1 = \infty$),* the curve indicates a completed agreement with the results of the standard form of the hydraulic jump usually used in a rectangular channel section, Eq.24. For more details, notice Table 1.

$$\frac{y_i}{y_j} = 0.5\left(\sqrt{1+8F_{rj}^{\;2}} - 1\right) \tag{24}$$

In many practical and designed cases the problem is to find the initial depth $\boldsymbol{y_1}$ for a given control depth $\boldsymbol{y_2}$ in the downstream of the jump. In this case the following model (Eq.23 b), will be used to provide the conjugate ratio $\boldsymbol{r}$, which depends on the relationship between Eqs. (10 , 14 & 16) and Eq.18. The solution of this model was achieved by trail and error method with helpful of the computer. However, all the results were represented in Fig.3 and Table 2.

$$A = B = \left(1 + 2.5k_2 + 1.5k_2^{\;2}\right) \tag{19 b}$$

$$C = \left(1 + k_2 - 3k_2\eta^2 - 3k_2^{\;2}\eta^2\right) \tag{20 b}$$

$$D = \left(-3\eta^2 - 6k_2\eta^2\right) \tag{21 b}$$

$$E = -3\eta^2 \tag{22 b}$$

$$\eta^2 = Fr_1^2 = \left(\frac{k_2+1}{rk_2+1}\right)^2 r^5 \left(\frac{k_2+1}{k_2+2}\right) F_{D2}^{\;2} \tag{22 C}$$

Therefore, Eq. 18 will be:

$$A\,r^4 + B\,r^3 + C\,r^2 + D\,r + E = 0 \tag{23 b}$$

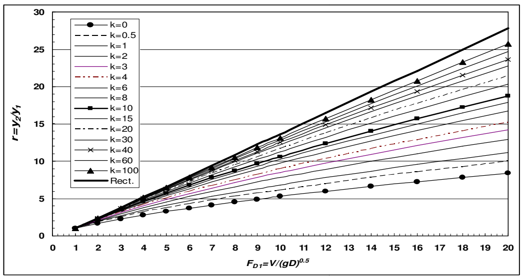

Fig.2: Family curves for the conjugate depths ratio r, corresponding to the upstream Froude number F_{D1} and k_1.

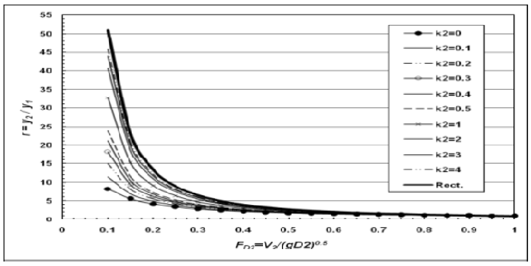

Fig.3: Family curves for the conjugate depths ratio r, corresponding to the Downstream Froude number F_{D2} and k_2.

It could be seen that, when F_{D2} is more than 0.5 the conjugate depth ratio *($r = y_2/y_1$)* has the same value for any section ratio k_2. For this reason the arrangement values of F_{D2} in Table (2) was concentrated on the low values of F_{D2}. Fig (4) shows the relationship between the upstream Froude number F_{D1} and the corresponding F_{D2} for varies values of k_1. The Figure indicates that when F_{D1} is greater than 20, the minimum value of F_{D2} approaches to 0.1 for the triangular section and 0.15 for the rectangular section. Indicating that the shape of the section has a little effect on the values of F_{D2} when F_{D1} is greater than 2 and has insignificant effect when the value of F_{D1} is less than 2.

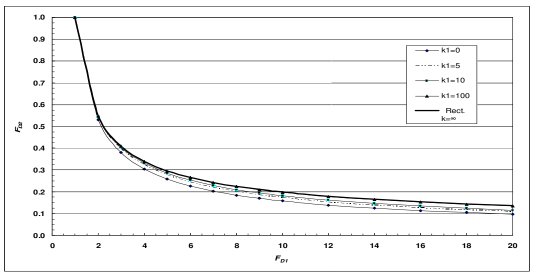

Fig.4: Relationship between F_{D1} and F_{D2} for varies values of k1.

F_{D1}	k1=0	k1=.5	k1=1	k1=2	k1=3	k1=4	k1=5	k1=6	k1=7	k1=8	k1=9	k1=10	k1=12	k1=15	k1=20	k1=30	k1=40	k1=60	k1= 100	Rect. k=∞	Rect. Eq.24
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.702	1.842	1.935	2.051	2.120	2.165	2.197	2.220	2.238	2.253	2.264	2.274	2.289	2.304	2.320	2.337	2.346	2.354	2.362	2.372	2.372
3	2.284	2.545	2.726	2.963	3.112	3.215	3.290	3.348	3.393	3.430	3.460	3.485	3.525	3.568	3.614	3.663	3.689	3.716	3.738	3.772	3.772
4	2.799	3.170	3.432	3.785	4.015	4.179	4.301	4.397	4.473	4.536	4.589	4.633	4.705	4.783	4.868	4.962	5.012	5.065	5.109	5.179	5.179
5	3.271	3.741	4.079	4.543	4.853	5.078	5.249	5.384	5.494	5.585	5.662	5.727	5.834	5.952	6.084	6.231	6.312	6.398	6.471	6.589	6.589
6	3.710	4.275	4.684	5.254	5.641	5.926	6.145	6.321	6.464	6.585	6.687	6.775	6.920	7.081	7.264	7.473	7.589	7.715	7.823	8.000	8.000
7	4.125	4.778	5.255	5.927	6.389	6.732	7.000	7.216	7.394	7.544	7.673	7.784	7.968	8.175	8.413	8.689	8.845	9.016	9.165	9.412	9.412
8	4.519	5.257	5.800	6.569	7.104	7.505	7.820	8.076	8.288	8.468	8.623	8.758	8.982	9.237	9.533	9.881	10.081	10.301	10.496	10.825	10.825
9	4.897	5.716	6.321	7.186	7.791	8.248	8.610	8.905	9.152	9.362	9.543	9.702	9.967	10.271	10.627	11.051	11.297	11.572	11.817	12.238	12.238
10	5.261	6.157	6.823	7.780	8.454	8.966	9.374	9.708	9.989	10.229	10.437	10.619	10.925	11.279	11.696	12.199	12.495	12.828	13.128	13.651	13.651
12	5.952	6.998	7.780	8.912	9.719	10.338	10.835	11.246	11.593	11.892	12.153	12.383	12.772	13.227	13.770	14.439	14.839	15.299	15.721	16.478	16.478
14	6.606	7.792	8.683	9.983	10.917	11.639	12.222	12.708	13.120	13.477	13.790	14.068	14.539	15.095	15.768	16.610	17.122	17.718	18.276	19.305	19.305
16	7.228	8.549	9.545	11.004	12.061	12.882	13.549	14.106	14.583	14.997	15.361	15.685	16.238	16.895	17.699	18.719	19.347	20.091	20.797	22.133	22.133
18	7.825	9.274	10.370	11.984	13.158	14.076	14.824	15.452	15.990	16.460	16.874	17.244	17.878	18.636	19.571	20.772	21.522	22.419	23.283	24.961	24.961
20	8.399	9.972	11.165	12.928	14.216	15.227	16.054	16.750	17.350	17.874	18.338	18.752	19.466	20.325	21.391	22.775	23.649	24.705	25.738	27.789	27.789

Table. 1. Conjugate depths ratio of a hydraulic jump in trapezoidal channel sections for a given y_1 with varies k_1.

F_{D2}	k2 = 0	k2 = 0.05	k2 = 0.075	k2 = 0.1	k2 = 0.15	k2 = 0.2	k2 = 0.25	k2 = 0.3	k2 = 0.35	k2 = 0.4	k2 = 0.45	k2 = 0.5	k2 = 0.55	k2 = 0.6
0.1	8.219	9.786	10.619	11.472	13.207	14.932	16.608	18.212	19.732	21.164	22.507	23.766	24.943	26.045
0.15	5.518	6.158	6.486	6.818	7.485	8.145	8.789	9.409	10.003	10.568	11.103	11.608	12.083	12.531
0.2	4.176	4.511	4.679	4.848	5.184	5.514	5.835	6.144	6.441	6.725	6.994	7.250	7.492	7.721
0.25	3.375	3.575	3.675	3.774	3.970	4.161	4.346	4.524	4.695	4.858	5.014	5.162	5.302	5.435
0.3	2.843	2.974	3.038	3.102	3.226	3.347	3.464	3.576	3.684	3.787	3.885	3.978	4.067	4.151
0.35	2.466	2.555	2.599	2.642	2.726	2.808	2.886	2.961	3.033	3.102	3.167	3.230	3.289	3.346
0.4	2.183	2.247	2.278	2.308	2.368	2.425	2.480	2.532	2.583	2.630	2.676	2.719	2.761	2.800
0.45	1.964	2.011	2.033	2.055	2.099	2.140	2.179	2.217	2.253	2.288	2.320	2.351	2.381	2.409
0.5	1.789	1.824	1.841	1.857	1.889	1.919	1.948	1.976	2.002	2.028	2.052	2.074	2.096	2.117
0.55	1.646	1.672	1.685	1.697	1.721	1.744	1.765	1.786	1.805	1.824	1.842	1.859	1.875	1.890
0.6	1.527	1.547	1.556	1.565	1.583	1.600	1.617	1.632	1.647	1.661	1.674	1.687	1.699	1.710
0.7	1.340	1.351	1.356	1.361	1.371	1.381	1.390	1.398	1.407	1.414	1.422	1.429	1.435	1.442
0.8	1.199	1.205	1.207	1.210	1.215	1.220	1.225	1.229	1.233	1.237	1.241	1.245	1.248	1.251
0.9	1.089	1.091	1.092	1.093	1.095	1.097	1.099	1.101	1.103	1.104	1.106	1.107	1.108	1.110
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table. 2. Conjugate depths ratio of a hydraulic jump in trapezoidal channel sections for a given y_2 with varies k_2.

F_{D2}	k2 = 0.7	k2 = 0.8	k2 = 0.9	k2 = 1	k2 = 1.25	k2 = 1.5	k2 = 1.75	k2 = 2	k2 = 2.5	k2 = 3	k2 = 3.5	k2 = 4	K2 = ∞ Rect.	Eq.24
0.1	28.041	29.793	31.336	32.702	35.499	37.637	39.311	40.649	42.638	44.031	45.053	45.830	50.984	50.981
0.15	13.350	14.074	14.718	15.290	16.472	17.382	18.098	18.673	19.532	20.136	20.580	20.919	23.181	23.181
0.2	8.142	8.518	8.854	9.154	9.778	10.264	10.648	10.958	11.423	11.751	11.994	12.179	13.431	13.431
0.25	5.681	5.902	6.100	6.278	6.651	6.943	7.176	7.364	7.649	7.851	8.001	8.115	8.899	8.899
0.3	4.307	4.447	4.574	4.688	4.929	5.118	5.270	5.394	5.581	5.715	5.815	5.891	6.421	6.421
0.35	3.450	3.545	3.630	3.707	3.870	4.000	4.104	4.190	4.319	4.413	4.482	4.536	4.906	4.912
0.4	2.873	2.939	2.999	3.053	3.168	3.260	3.334	3.395	3.488	3.556	3.606	3.645	3.922	3.922
0.45	2.462	2.509	2.552	2.591	2.675	2.741	2.795	2.840	2.909	2.958	2.996	3.025	3.234	3.233
0.5	2.155	2.190	2.222	2.250	2.312	2.361	2.401	2.434	2.485	2.523	2.551	2.573	2.732	2.732
0.55	1.919	1.945	1.968	1.989	2.035	2.072	2.102	2.127	2.166	2.194	2.215	2.232	2.355	2.355
0.6	1.731	1.751	1.768	1.784	1.818	1.846	1.869	1.888	1.917	1.938	1.955	1.967	2.062	2.062
0.7	1.453	1.464	1.474	1.483	1.502	1.517	1.530	1.540	1.557	1.569	1.579	1.586	1.642	1.642
0.8	1.257	1.263	1.268	1.273	1.282	1.290	1.297	1.303	1.311	1.318	1.323	1.327	1.357	1.357
0.9	1.112	1.114	1.116	1.118	1.122	1.125	1.128	1.130	1.134	1.136	1.138	1.140	1.153	1.153
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table. 2. Continued

Jump Characteristics

The characteristics of the hydraulic jump in horizontal trapezoidal channel sections represented by some of terminologies will be discussed below.

Energy Dissipation Efficiency

Hydraulic jumps have been widely used for energy dissipation in hydraulic constructions. Many researchers have paid their attention to them for a long time, (Hashmi, 2003) & (Chaudhry, 2008). The hydraulic jump naturally dissipates energy through turbulence, which can be highly erosive if proper channel protection is not installed, (Hager, 1992). It is therefore preferable, when a hydraulic jump is expected, to control the size and location of the jump in order to localize energy dissipation and erosion, (Stahl and Hager, 1999). The energy loss due to the hydraulic jump is equal to:

$$\Delta E = E_1 - E_2 \qquad (25)$$

With

$$E = y + \frac{V^2}{2g} \qquad (26)$$

Where:

ΔE: Energy loss due to the jump.
E_1: Specific energy before the jump.
E_2: Specific energy after the jump.

The ratio of *(E_2 / E_1)*, represents the efficiency of the jump, *(Ef)*, so:

$$Ef = \frac{E_2}{E_1} \qquad (27)$$

Therefore, the relative losses is equal to:

$$\frac{\Delta E}{E_1} = 1 - \frac{E_2}{E_1} \qquad (28)$$

The difference between the conjugate depths is the height of the jump h_j, and the ratio h_j/E_1, represents the relative height:

$$\frac{h_j}{E_1} = \frac{y_2}{E_1} - \frac{y_1}{E_1} \qquad (29)$$

Where:

y_1/E_1: Relative initial depth.
y_2/E_1: Relative sequent depth.

It is important to express all the above ratios in term of dimensionless functions of F_{D1}. Depending on Eq.26 and using Eqs.(6 & 10), the relative initial depth could be expressed as:

$$\frac{y_1}{E_1} = \frac{2(k+2)}{2(k+2)+(k+1)\ F_{D1}^2} = \frac{2}{2+F_{r1}} \qquad (30)$$

So, the relative sequent depth will be:

$$\frac{y_2}{E_1} = \frac{y_1}{E_1}\ r \qquad (31)$$

Applying Eq.26 at the downstream of the jump, considering Eqs. (14 to 16), results:

$$\frac{E_2}{y_1} = r + \frac{(k+1)^3}{2r^2(k+2)(k+r)^2}\ F_{D1}^2 \qquad (32)$$

Consequently, from Eqs. (30 & 32), the efficiency will take the following form:

$$\frac{E_2}{E_1} = \frac{2(k+2))}{2(k+2))+(k+1)\ F_{D1}^2}\ x \left[r + \frac{(k+1)^3}{2r^2(k+2)(k+r)^2}\ F_{D1}^2 \right] \qquad (33)$$

It should be remembered that, the value of r in the above equations, represents the solution of Eq.23a corresponding to the values of F_{D1} and k_1. Since the efficiency and the other relative's definitions become functions of F_{D1}, plotting them against Froude number produces set of chrematistic curves for various values of k_1, see Fig.5.

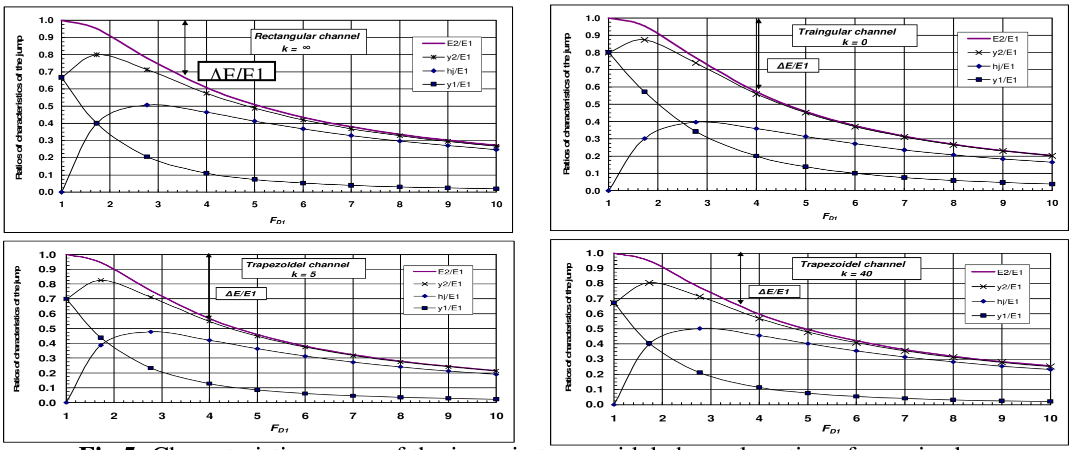

Fig.5: Characteristic curves of the jump in trapezoidal channel sections for varies k_1.

The figure indicates that the maximum y_2 / E_1 always occurs at $F_{D1} = 1.73$, independent on the shape of the section k_1, within a range of *0.874 to 0.8 for* (k_1 *= 0 to* ∞) respectively, giving a maximum value in triangular shape. The maximum h_j / E_1 is always at $F_{D1} = 2.78$, independent on the shape of the section k_1, within a range of *(0.4 for k_1 = 0 to 0.5 for k_1 = ∞),* giving a minimum value in triangular shape, see Fig. (6). Also, since E_1 increases when F_{D1} increases, the relative height h_j/E_1 tends to decrease when F_{D1} increases. However, it should be noted that the decreasing of h_j/E_1 does not mean a decreasing of y_1 or y_2 which are expected to increase due to the increasing of the discharge at the higher F_{D1}.

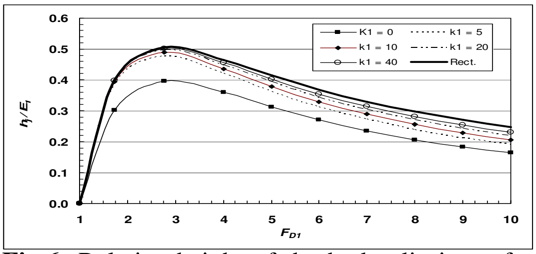

Fig.6: Relative height of the hydraulic jump for various trapezoidal channel shapes, k_1.

Fig.5 shows that the value of y_1 / E_1 at $F_{D1} = 1$, is equal to 0.67 for $k_1 = \infty$ and 0.8 for $k_1=0$, while it varies from 0.67 to 0.8 for trapezoidal sections. These results could be explained as follows:

When F_{D1}=1, the upstream depth y_1 is a critical depth *(Yc)* and consequently E_1 reduces to the minimum specific energy E_{min}. Therefore:

$$\frac{y_1}{E_1} = \frac{Yc}{E_{\min}} = \frac{Yc}{Yc + \frac{Vc^2}{2g}} \qquad (34)$$

Where *Vc* is the critical velocity.

The criteria of critical flow condition is, (Chaudhry, 2008).

$$\frac{Vc^2}{2g} = \frac{D}{2} \qquad (35)$$

Or

$$\frac{y_1}{E_1} = \frac{Yc}{E_{\min}} = \frac{Yc}{Yc + \frac{D}{2}} \qquad (36)$$

From the background of the hydraulic channel, the hydraulic depth *D*, is equal to (*y* and *y/2*) in rectangular and triangular sections respectively. Hence, Eq.36 provides a value of (2/3) and (0.8) in rectangular and triangular sections respectively.

Furthermore, consider Eqs. (8 & 9) for the hydraulic depth *D* in trapezoidal shape, Eq.36 could be expressed as:

$$\frac{y_1}{E_1} = \frac{Yc}{E_{\min}} = \frac{2k + 4}{3k + 5} \qquad (37)$$

Which also indicates that in case of a trapezoidal section, the ratio Y_C / E_{min} is between 4/5 for a triangular shape *(k_1=0)* and 2/3 for a rectangular shape *(k_2= ∞)*, while it depends on the values of *k* in the other shapes of trapezoidal section. So, Eq.37 could be considered as a general formula to estimate the value of Yc/E_{min} in trapezoidal section corresponding to the section ratio k_1.

Fig.7 shows the efficiency of the hydraulic jump in trapezoidal channel sections. The figure indicates that the section ratio k_1, has insignificant effect when F_{D1} is less than 3. Also, when F_{D1} is grater than 10, the efficiency sustain at a constant value in a range of 73 to 80 percent corresponding to k_1-value. However, in spite of that the rectangular section has a minimum efficiency corresponding to the other sections; the other shapes do not increase the efficiency higher than ten percent, which is insignificant value comparing to the difficulties of the constructions of a triangular or trapezoidal channel. Hence, practically speaking, the rectangular section could be considered more suitable section in the design of the energy dissipation structures.

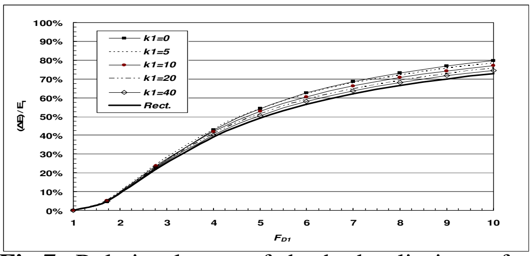

Fig.7: Relative losses of the hydraulic jump for various trapezoidal channel shapes, k_1.

The analysis indicates that in case of $F_{D1} > 6$, the efficiency curve *(E_2/E_1)* tends to be asymptote to the sequent relative curve *(y_2/E_1)*, independent on the section factor k_1, see Fig.8. Also, the figure shows that when k_1 is grater than 10, the curves join together to a constant value for all values of F_{D1}. This fact could be explained as follows:

Based on the results of the Fig.7, the velocity after the jump is always decreased due to the increasing of the efficiency where the flow losses the most energy through the jump when $F_{D1} > 6$, *(steady or strong jump)*. At the same time, the sequent depth is still increasing, note Fig.2. Consequently the remaining specific energy after the jump is essentially due to the sequent depth y_2. Therefore, when $F_{D1} > 6$, the velocity head after the jump could be neglected and the specific energy will be estimated by the sequent depth only. In other words, $E_2 = y_2$ for $F_{D1} > 6$.

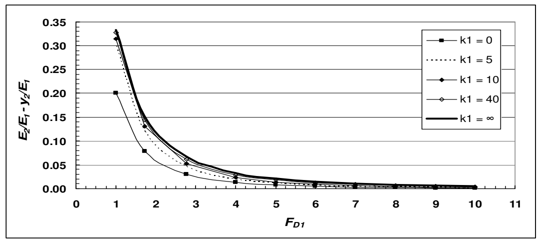

Fig.8: The effect of F_{D1} on the specific energy - sequent depth relationship.

Hydraulic jump length

The length of the hydraulic jump is generally measured to the downstream section at which the mean water surface attains the maximum depth and becomes reasonably level, (Philip, 2006). The length of the hydraulic jump is typically obtained from empirical functions of the jump height, based solely upon experimentation (Sturm, 2001). and the location depends on both the length and height of the jump, as well as, the upstream and downstream water surface profiles Chow (1994). Mohd (2008), drove the following differential equation to determine the jump ordinate H at known values of n, H_2 *and* F_{r1}.

$$\left[1-\frac{1}{H_2}\right]^2 \frac{(1+n)(1+nH_2+n)(1+2nH)}{H^2(1+nH_2)(1+nH)^2}\frac{\partial H}{\partial \xi} = \frac{(1+n)}{(1+nH)}H\left[-1+\frac{(1+n)}{(1+nH)}H\right]+\frac{1}{3F_{r1}^2}\left[\frac{H(3+2nH)}{2(1+nH)}-\frac{(3+2n)}{2H(1+nH)}\right]$$

With $n = \frac{1}{2k}$ and $\zeta = \frac{x}{\varepsilon \; y_2}$ (38)

where

$\boldsymbol{\varepsilon}$**:** universal constant for eddy kinematic viscosity, independent of channel geometry.

ζ*:* non-dimensional constant *(= $x/\varepsilon y_2$)*.

H: ordinate of jump profile *(= y/y_1)*

H_2: sequent depth ratio *($r = y_2/y_1$)*

In this study, the solution of Eq.38 was provided using *Runge-Kutta* method to determine the length of the jump at known values of k_1, *r and* F_{r1}, see Fig.9.

Also, AFZAL (2002). developed the following model to express the length of the hydraulic jump (L_j) in trapezoidal channel sections.

$$\frac{L_j}{y_2} = \varepsilon\,(1-\alpha)\,\Delta \tag{39}$$

$$\Delta = \frac{4K_1^2 K_2}{f(\omega_m)+B} \tag{39 a}$$

$$f(\omega_m)+B = \begin{bmatrix}(7+32\alpha+41\alpha^2+32\alpha^3+7\alpha^4)M^3+12\alpha(1+\alpha)^3M^2 \\ +\alpha^2(41+74\alpha+41\alpha^2)M+18\alpha^3(1+\alpha)\end{bmatrix}/4 \tag{39 b}$$

$$K_1 = M\,(1+\alpha)+\alpha \tag{39 c}$$

$$K_2 = 2M(1+\alpha+\alpha^2) + 3\alpha(1+\alpha) \quad \text{(39 d)}$$

With

$$M = \frac{zy_1}{b} = \frac{1}{k_1}, \alpha = \frac{1}{r} \textbf{ and } \varepsilon \approx 2.578 \quad \text{(39 e)}$$

Fig.9 explains a comparison between the results of Eqs. (38 & 39) and the experimental work of USBR for rectangular section and (Argyropoulous, 1961). for triangular section. The comparison shows that the results due to the model of Eqs. (39) are more precise and applicable than the results of Eq.38. Hence, the model of Eqs.39 was considered here to estimate the length of the hydraulic jump in trapezoidal channel.

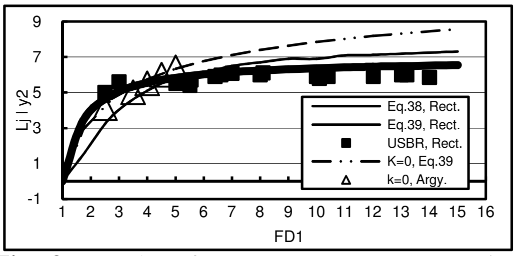

Fig. 9: Results of Eqs. (38 & 39), Comparing with other experimental works.

Based on the model of Eqs.39 and depending on the solutions of Eq.18 in Table 1, the length of the jump in trapezoidal channel sections were estimated and the results prepared in the dimensionless charts of Figs. (10 & 11). The charts show that for a large value of F_{D1} the jump length L_j/y_2 is independent on the upstream Froude number neither less the value of k_1. For the rectangular shape, the results indicate that when F_{D1} reaches to a very high values, the jump length Lj/y_2, is practically constant at approximated value of 6.9. This is because in case of a rectangular shape where M = 0, Eq.39a reduces to Δ=2.667. Consequently the term (ξ x Δ) in Eq.39 becomes 6.9. At the same time when F_{D1} approaches to infinity, r approaches to infinity too and $\alpha = 0$, which makes Eq.39 to give 6.9. It should be said that (Subramanya, 1998). and (Elevatorski, 1959). proposed the constant 6.9 but for $F_{D1} > 5$. In this study, when F_{D1} = 5, the jump length L_j/y_2 is about 5.83 which indicates a difference of 17 percent.

Also, the results indicate that for a constant Froude number F_{D1}, the jump length ratio is proportional with the section factor k_1 until a value of k_1 between 3 to 4. After that (*for* k_1 > 4), the relation will be decreased asymptotic to a constant value, see Fig.12. That means, the maximum ratio (L_j/y_2), is always near a section ratio of $k_1 \approx 3$ to 4, independent on the Froude number F_{D1}. Therefore, for purposes design it is recommended to avoid this ratio in order to minimize the jump length.

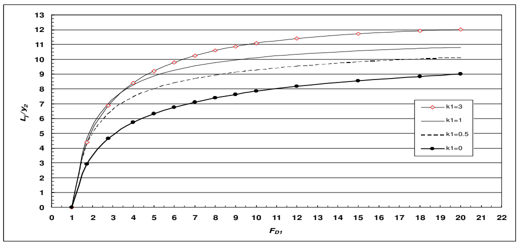

Fig. 10: Hydraulic jump length-Froude number relationship for k_1= 0 to 3.

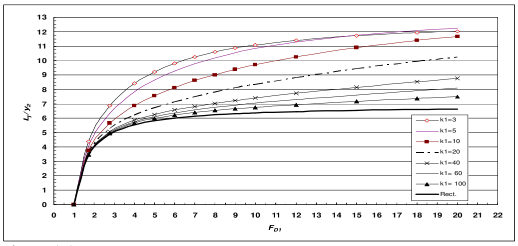

Fig. 11: Hydraulic jump length-Froude number relationship for k_1= 3 to ∞.

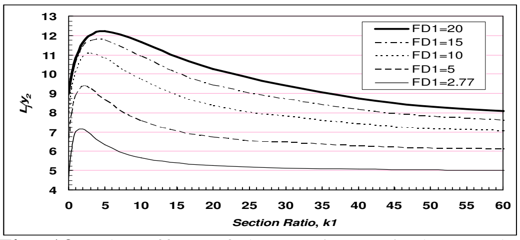

Fig. 12: The effect of the section ratio k_1, on the maximum length of the hydraulic jump.

Conclusions

Applying the momentum conservation across a hydraulic jump in trapezoidal channel sections produced a general fourth order polynomial equation which provides a conjugate depths ratio of arbitrary cross sections. The solution was provided using *Newton-Raphson* method, and the results are represented as a dimensionless charts and Tables. When the values of the upstream Froude number F_{D1}, are less than 2, the differences between the conjugate depths ratios have low significant change for all the shapes. The maximum values of y_2 / E_1 and h_j / E_1 always occur at F_{D1} = *1.73* and F_{D1} = *2.78* respectively, independent on the shape of the section (k_1). When F_{D1} is greater than

6, the velocity head after the jump could be neglected, (*i.e.* $E_2 = y_2$). The type of cross section has a little effect on the values of F_{D2} for $F_{D1} > 2$ and insignificant effect when F_{D1} is less than 2. The minimum values of F_{D2} for all sections range from 0.1 in triangular section to 0.15 in rectangular section, which is insignificant range. Even though, the energy dissipation efficiency of the hydraulic jump indicates that non-rectangular sections are more efficient in high Froude numbers, but these sections produce longer jumps, stability problems, and difficult in constructions. Therefore, from the hydraulic and structural point of view, the rectangular section is the preferable one in the design of hydraulic structures. Moreover, neither less of F_{D1}, the maximum ratio of jump length (Lj / y_2), always occurs when the section ratio is about $k_1 \approx 3$ to 4, which is recommended to avoid that for no longer jump.

Nomenclature

A_1 & A_2: Cross-sectional area before and after the jump, respectively.
b: Bottom width of the sectional area.
k: section ratio
E_1: Specific energy before the jump.
E_2: Specific energy after the jump
Ef: jump efficiency
E_{min}: Minimum specific energy.
F: Specific force
F_r: Froude number in term of the depth of flow y.
F_D: Froude number in term of the hydraulic depth $D = A/T$.
g: Gravity acceleration force.
H: ordinate of jump profile ($= y/y_1$).
H_2: sequent depth ratio ($r = y_2/y_1$).
L_j: the length of the hydraulic jump
Q: Flow rate
r: Conjugate depths ratio of the initial and sequent depths, (y_2 / y_1) .
T: Top width of the sectional area.
V: Mean velocity.
y_1/E_1: Relative initial depth.
y_2/E_1: Relative sequent depth
Yc: Critical depth.
z: side slope
Z_{C1} & Z_{C2}: Distances of the centroids sections from the free surface area before and after the jump, respectively
ΔE: Energy loss due to the jump.
ε: universal constant for eddy kinematic viscosity, independent of channel geometry.
ζ: non-dimensional constant ($= x/\varepsilon y_2$).

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الملخص العربي

خصائص القفزة الهايدروليكية في المقطع الشبه منحرف

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الجامعة المستنصرية / كلية الهندسة / قسم هندسة البيئة

الملخص:

في هذا البحث، تم تحليل ومناقشة خصائص القفزة الهيدروليكية في المقطع الشبه منحرف وتم اشتقاق نموذجا رياضيا شاملا لحساب مسائل القفزة الهيدروليكية لجميع حالات المقطع الشبه منحرف (مستطيل, مثلث والشبه منحرف) بالاعتماد على مبدأ الزخم. لقد تم استحصال الحلول والنتائج للنموذج الرياضي بالاعتماد على الحل العددي وبواسطة طريقة نيوتن رافسن للمماس. تم عرض النتائج في جداول ومرتسمات تبين العلاقة بين نسبة عمقي القفزة الهيدروليكية $(r = y2 / y1)$ وقيم رقم فرود وكذلك كفاءة القفزة في تشتيت الطاقة، والعلاقة بين قيم رقم فرود قبل وبعد القفزة ($FD1$ and $FD2$) لمدى واسع من معامل المقطع $(k=b/zy)$. بواسطة النموذج الرياضي تم تخمين طول القفزة الهيدروليكية بالاعتماد على نسبة عمقي القفزة $(r = y2 / y1)$ ولمدى واسع من معامل المقطع $(k=b/zy)$ أيضا. لقد بينت النتائج إن شكل المقطع ليس له تأثير معنويا على كفاءة القفزة الهيدروليكية في تشتيت الطاقة على الرغم من إن الطاقة المشتتة في الشكل المثلث تكون أكثر بحدود ١٠% في حالة قيم $FD1$ العالية. بالإمكان إهمال السرعة في مؤخرة القفزة عندما يكون رقم فرود في مقدم القفزة $FD1$ أعلي من ٦. عندما يكون معامل المقطع بحدود ٣ فان نسبة طول القفزة إلى العمق بعد القفزة $(Lj / y2)$ يصل إلى أعلى مستوياته غير معتمد على قيمة رقم فرود في مقدم القفزة $FD1$. عند مقارنة النتائج المستحصلة من الموديل الرياضي مع نتائج دراسات عملية أخرى اتضح توافقا جيدا جدا في جميع الحالات.