Abstract:
In karst aquifers, temperature distribution play an additional important role since they carry information about internal aquifer structures. The aim of the present work is to develop a two dimensional heat transfer model in a karst aquifer. Navier Stokes equation is used to simulate the groundwater velocity in the conduit system where the porosity tends to one, and means water velocity was taken into account in the fractured rock. Heat transport equation was applied to simulate the temperature distribution in a karst aquifer, and k-ε turbulent model is used to simulate the turbulent viscosity. The model was applied to the karst system of Val d'Orléans. Temperatures are measured in thirteen wells with different depth in 29 Jun 2011. Results have shown that the model was not sensitive to the variation of water density, but it is sensitive to the variation of specific heat and density of the rock especially in the fractured system. Also, the model was varied sharply with the velocity of water in sinkhole points, and any variation in the depth of saturated zone. The comparison between measured and calculated temperatures in wells is very good.

Key word: Karst aquifers, Heat transport, Conduit and diffuse flow systems, Numerical model and Val d'Orléans

Introduction:
Karst forms when groundwater dissolves pockets of limestone, dolomite, or gypsum in bedrock. This dissolution process increases the bulk permeability of the massif, developing a conduit network of high hydraulic conductivity, with short water residence time, and preserving micro fractured blocks with long water-residence time (Dogwiler et al., 2007). Thus, karstification provokes flow heterogeneity, increasing the permeability contrast between conduit flow and diffuse flow systems. Karst system is mainly characterized by four elements. The first is sinkholes which recharge the karst system. The second is the underground drainages or conduits which are largely influenced by sinkholes and consequently the water flow in these regions is high. The third is fractured system (diffused system) which is weakly influenced by sinkhole and consequently the water flow in these regions is slow. The last is spring point in which the water is emerged at the surface. In this context, karst systems are highly vulnerable compared to other groundwater systems, since potential contaminants can easily reach the groundwater (Genthon et al., 2005; O’Driscoll and DeWalle, 2006; Dogwiler et al., 2007).

The use of heat as a groundwater tracer, in contrast to the use of chemical tracers, is attractive because of the ease of measuring temperature with high precision (errors as low as ±0.03 °C). Groundwater temperatures are influenced by the temperature of recharge, mixing of different waters resulting from groundwater flow. (Andrieux, 1978; Crowther and Pitty, 1982; Roy and Benderitter, 1986; Lastennet, 1994; Martin and Dean, 1999; Birk et al., 2004). have used water temperature jointly with other natural hydro dynamical and hydro chemical responses, as additional information to characterize the different flow types and the structural organization of drainage patterns in karst aquifers. Groundwater applications have been developed to model quick-flow in karst conduits, diffuse flow in fractured and, and the
interaction of these two flow regimes. Fluid flow and solute/heat-transfer numerical models that include both of these flow regimes include (Benderitter et al., 1993; Liedl and Sauter, 2000; Birk, 2000; Andre and Rajaram, 2005; Birk et al., 2004). With these distributed-parameter models, velocities are estimated from the flow simulation and then are used in the transport simulation. Additional insight into general heat-transfer theory for pipe and channel flow is described by (Gnielinski, 1976; Aravinth, 2000; Beek et al., 1999; Benim et al., 2004). As the conduits are highly influenced by the contamination of rivers (as the water of sinkholes), any information on conduit locations usually is unavailable. For cases where wells or springs have a temperature response that is influenced by conduit flow, the conduit network is globally defined. This paper presents a two-dimensional numerical water flow/heat transport model that is explored as an alternative that might be useful to locate the conduit networks in the karst system of the Val d’Orléans. This model simulates the temperature response to recharge in wells and assumes that wells receive at least some of its water from a nearby conduit. The water flow will be simulated in conduit system by Navier-Stokes equation, but the model does not simulate the water flow in the fractured system (in which the permeability is less than that in the conduit system). The water velocity in the fractured system will be carried out as mean velocity. The results of the model will be verified with temperatures observed in the wells. The viscosity gradient will be calculated by using K epsilon turbulent model.

Characteristics of the experimental field area:
The karst aquifer of the Val d’Orléans is the largest in France in terms of flow rate (10 m$^3$/s) and provides the mean water resource of the Orléans city (Albéric and Lepiller, 1998). The Val d’Orléans is considered as a vast depression of the major bed of the Loire river, 37 km long and from 4 to 7 km wide (Fig. 1). The karst aquifer is hosted within an Oligocene carbonate lacustrine deposit occurring in the center of the Paris basin and called the limestone of Beauce (Guillocheau et al., 2000). This latter formation display variable repartition with a significant primary porosity except for micritic facies, this porosity is increased by karstification leading to a relative high permeability (5E-10 to 2E-9 m2) at hectometric scale (Martin and Noyer, 2003). The latter is overlapped by the quaternary alluvia of the Loire river.
The Loire river feeds more than 85% of the water hosted in the carbonated karstic aquifer. The estimated inflow of the Loire river in the sinkhole infiltration area of Jargeau varies from 15 to 20 m$^3$/s and it can reach 100 m$^3$/s during floods (Zunino, 1979; Chéry, 1983; Lepiller and Mondain, 1986). Karst networks are well known in the left bank of the Loire river. The water runs from Jargeau through the karst conduits networks towards the direction of the springs of the Loiret river, (Zunino, 1979; Chéry, 1983; Lepiller and Mondain, 1986), as shown in figure (1). The springs of Loiret river are called the Bouillon and the Abîme, they are considered as the main emergences of the water lost close to Jargeau in the Loire river (from 0.3 to 5 m$^3$/s). The mean aquifer outflow is an underground emergence in the Loire river located around the confluence of Loire - Loiret. Previous studies showed the relation between these springs and the sinkholes points at Jargeau within the Loire river using dye tracer tests (Zunino, 1979; Chéry, 1983; Albéric and Lepiller, 1998; Lepiller, 2001; Albéric, 2008). The main karstic conduits were located according to the depressions of the piezometric surface and to the different connections identified by the tracer tests presented in figure (1).

Figure (1): Underground waters karstic circulations of the Val d’Orléans city (Albéric and Lepiller, 1998).
**Governing equations:**

Numerical simulations of fluid flow and heat transport in a karst aquifer were used to investigate the temperature distribution in the karst and by consequence to determine the karstification degree of the karst aquifer. In the present work, Navier Stokes equation is applied to simulate the water velocity in conduit system, as a result to the grand porosity in this system. A uniform velocity is taken in the fractured rock system where the porosity is highly less than that in conduit system. To determine the temperature distribution in the karst, heat transport equation is used. Due to the variation of the temperature in the karst system, the viscosity will be changed, and to calculate this variation, K epsilon turbulent model is used.

Navier Stokes equation for two dimension is:

\[
\rho_w \frac{\partial \mathbf{u}}{\partial t} + \rho_w \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \mathbf{q} + \rho_w \mathbf{g}
\]

Heat transport equation in the karst for two dimension is:

\[
\frac{\partial}{\partial t} \left[(1-\phi) \rho_w C_T \frac{\partial T}{\partial t} + \phi \rho_w \mathbf{C}_v \mathbf{T} \right] + \phi \rho_w \mathbf{C}_v \nabla \mathbf{T} = \lambda \nabla^2 \mathbf{T}
\]

K epsilon turbulent model for two dimension is:

\[
\rho_w \frac{\partial \varepsilon}{\partial t} + \rho_w \mathbf{u} \cdot \nabla \varepsilon = \nabla \cdot \left( \frac{\mu_t}{\sigma_t} \nabla \varepsilon \right) + C_1 \varepsilon \frac{\varepsilon}{k} - G - \rho_w \mathbf{E}
\]

\[
\rho_w \frac{\partial k}{\partial t} + \rho_w \mathbf{u} \cdot \nabla k = \nabla \cdot \left( \frac{\mu_t}{\sigma_t} \nabla k \right) + C_2 \varepsilon \frac{\varepsilon}{k}
\]

To calculate the turbulent viscosity, the following equation is used:

\[
\mu_t = C_{\mu} \rho_w \frac{k^2}{\varepsilon}
\]

Where:

- \(\rho_w\) is the water density,
- \(q_w\) is water velocity vector,
- \(t\) is the time,
- \(p\) is the water pressure,
- \(g\) is the acceleration gravity,
- \(\phi\) is the porosity of the karst system,
- \(\rho_r\) is the rock density,
- \(C_r\) is the specific heat of the rock,
- \(C_w\) is the specific heat of the water,
- \(T\) is the water temperature,
- \(\lambda\) is the heat conductivity,
- \(k\) is the turbulence kinetic energy,
- \(\varepsilon\) is the dissipation rate of turbulent kinetic energy,
- \(G\) is the production of turbulence kinetic energy,
- \(\sigma_k\), \(\sigma_\varepsilon\), \(C_1\), \(C_2\), \(C_{\mu}\) are constants. Les valeurs des constantes sont (Leschziner et Rodi, 1983).

\[ C_{\mu} = 0.09, \ C_1=1.44, \ C_2=1.92, \ \sigma_\varepsilon =1.3, \ \sigma_k=1 \]

In the present work, the variation in the density of water and rock can be calculated from equations (6) and (7), respectively. The variation in the specific heat of water and rock can be calculated from equation (8) and (9), respectively (Somerton, 1992; Douglas and Jacob, 2004).

\[ \rho_w(T) = 1043.196 - 42.966623 \exp{(0.006895T)} \]  \( (6) \)

\[ \rho_r(T) = \frac{2650}{1 + (T - 20) \times 0.5 \times 10^{-4}} \]  \( (7) \)

\[ C_w(T) = 0.0002374 + 8.06817 \times 10^{-6} - 8.03671 \times 10^{-14} T \]  \( (8) \)

\[ C_r(T) = 1234.257 - 454.546 \exp{(-0.0039733 T)} \]  \( (9) \)

**Heat transport in the karst system of the Val d’Orléans:**

Karst system of the Val d’Orléans has many sinkhole points which are located on the Loire river at the city of Jargeau, and it has many spring points as shown in figure (1). In this work, the temperature is monitored at thirteen wells and one spring point located in the karst system of the Val d’Orléans. In general, water in the conduit includes sinking river water and diffuse flow (from fractured system) entering the conduit along its length. In addition to water from the conduit, a well or spring also might receive local diffuse flow that has not interacted with the conduit. For example, a well that is south of the conduit may induce flow from the conduit and also from diffuse flow within the well’s zone of influence on the north, south, and east sides of the well (Fig. 2) and consequently it can be observed a variation in the water temperature of the well. But in the most cases, it can be observed many wells in which the temperature is constant. This can be attributed to the location of the well, the variation of the water temperature in the well decrease when the well far away from the conduit and vice ve
Figure (2): Schematic diagram of a karst system

Depending on the previous description, the temperature is monitored at thirteen wells and one spring point located in the karst system of the Val d'Orléans. Position of wells in the calculational region is illustrated in Fig. (3). The temperature measurements in wells are shown in figures (4, 5 and 6). These measurements are provided in 29/06/2011 when the temperature of Loire river and the Bouillon spring were 26.5 °C and 15.6 °C, respectively. Figure (4) shows that the water temperature in wells of Ligne, Piezometre, and Moret is nearly stable but in the well of Moret 2, the temperature decreases 3 °C starting from the depth of 12 m. This variation in the temperature can be attributed to the water coming from the conduit system. The variation of groundwater temperature in wells of Boires 1 and Boires 2 is greater than that in the well of Moret 2, as shown in figure (5).

Figure (3): Wells location in the karst system of the Val d'Orléans

In these wells, the variation reaches to 12 °C, this means that these wells are close to the conduit system. But the temperature is stable in wells of Ligerienne and Ormeaux. Figure (6) shows the wells of Berruet 1 and Berruet 3 are affected by the conduit system but less than that in wells of Boires 1 and Boires 2.

Figure (4): Water temperatures measurements in wells of Ligne, Piezometre, Moret 2, and Moret

Figure (5): Water temperatures measurements in wells of Ligerienne, Boires 1, Ormeaux, and Boires 2.

Figure (6): Water temperatures measurements in wells of Berruet 1, Berruet 3, Berruet 4, Berruet 6, and Berruet 7
Mathematical modeling:
The study area in the karst aquifer of the Val d'Orléans starts from Jargeau (where the
sinkholes on the Loire river are existed) to the
last spring point on the Loiret river. The study
area is considered as a rectangular area with
the length 21000 m and the width 4000m. Two
dimension numerical model is carried out to
simulate the water temperature distribution in
the karst system of the Val d'Orléans. The
porosity in conduit system and in the fracture
time rock system is 90% and 10% respectively.
The pathway of the conduit system suggested in
the present research is shown in figure (7).
This pathway is suggested according to
(Lepillier, 2001; Albéric, 2008).

Initial conditions constitute values of
velocity, temperatures, density and specific
heat of the water and rock, water viscosity,
turbulent kinetic energy and dissipation rate of
turbulent kinetic energy. Concerning the
velocity, it is carried out the velocity value
measured during the summer season. Then the
water velocity inlet to the conduit system is 75
m/hr, this velocity is varied in the conduit
system according to Navier Stokes equation,
but it is constant in the fractured rock matrix.
The velocity inlet to the fractured rock system
is a half of previous velocity. Initially the
temperature in the study area is that measured
in the Bouillon spring on 29 Jun 2011, it was
15.6 °C, expect on the sinkhole points on the
Loire river in which the initial water
temperature is that measured on Loire river, it
was 26.5 °C on 292011. The initial values for
the density and specific heat of the water and
rock are calculated from equations (6, 7, 8,
and 9). The initial values of water viscosity,
kinetic energy and dissipation rate of turbulent
kinetic energy are obtained by the following
equations:

\[ \mu = 0.077 \rho U h \quad \ldots (10) \]
\[ \varepsilon = S q \quad \ldots (11) \]

Where: h is the mean water depth (depth of
saturated zone in the karst aquifer), S is the
piezometric of the water slope, U is the
friction velocity which is equal to \( \sqrt{ghS} \). The
piezometric of the water slope is calculated in
each region in the study area according to the
piezometric map provided by (Zunino, 1979).
Equation (5) is used to calculate the initial
value of turbulent kinetic energy.

Boundary conditions of the study area are
illustrated in fig (8). The finite differences
technique is used to solve partial differential
equations in the present numerical model. The
length and width increments are 5 m. Also, the
final time of the model is three months and the
time step is 5 min, and the thermal
conductivity is 1.3 J/sec.m. °C.

Results and discussions:
Many parameters influence on the water
temperature distribution in a karst aquifer, as
the depth of saturated zone, water velocity,
viscosity and density effects, porosity, density
and specific heat of the rock. Therefore, it was
important to study the effect of the variations
of these parameters separately to describe the
rate and pattern of heat transport and prioritize
their influences.

Neglecting the density difference between
the temperature of Loire river and
groundwater temperature is carried out to
study the effect of density on the temperature
distribution, and keeping a constant density
during a time period of study equal to initial groundwater density. A comparison between isotherms with and without density effect is shown in fig (9). It can be clearly observed, all isotherms are not influenced by the change of water density. This due to the small temperature difference between Loire river temperature (26.5 °C) and groundwater temperature (15.6 °C). To investigate the effect of the variation of water slope along the study reach which is coming from the piezometric map, a constant water slope along the study reach is taken into account. From fig (10), it can be observed that the water slope parameter influences on the behavior of temperature distribution. When the water slope is varied, the distribution of temperature levels advances more in transverse direction as that when the water slope is constant.

The water velocity in sinkhole points on Loire river has a great effect on the behavior of temperature distribution along the study reach. As shown in fig (11), all isotherms are advanced longitudinally and transversely with any increase in the water velocity values. This phenomenon can be attributed to the effect of advective term in the heat transport equation, which is responsible for the advance of isotherm along the study reach. Fig (12) shows the effect of water depth in the saturated zone. According to Albéric and Lepiller (1998), the mean depth of saturated zone for the karst system of the Val d’Orléans is 25 m. Any decrease in the depth of saturated zone causes a retardation of the temperature isotherms along the study reach, as shown in fig (12). This can be attributed to the effect of the depth of saturated zone on the friction velocity and water viscosity and by consequence on the temperature distribution. In order to show the effect of the variation of the specific heat and the density of the rock on the behavior of the temperature distribution, equations (7) and (9) are neglected. This means that the specific heat and the density of the rock are constant in the calculations. In the case of the specific heat and the density of the rock are constant, all isotherms are retarded in the transverse direction, but they are advanced in the longitudinal direction, as shown in fig (13). This may be due to the effect of the specific heat and the density of the rock on the domain of fractured system in the karst aquifers only.
Conclusions

In karst aquifers, temperature signals play an additional important role since they carry information about internal aquifer structures. A two dimension heat transport numerical model was developed to simulate the temperature distribution in a karst aquifers composed conduits and fractured systems. The model was based on the Navier Stokes equation to simulate the groundwater velocity in the conduit system where the porosity tends to one, heat transport equation to simulate the temperature distribution in a karst aquifer, and finally k-ε turbulent model to simulate the turbulent viscosity.

The model was applied to the karst system of the Val d'Orléans. This system is very developed in which there are many sinkhole points on the Loire river and many spring point along the Loiret river. Temperatures are measured in thirteen wells with different depth in 29 Jun 2011 when the temperature of Loire river and the Bouillon spring were 26.5 °C and 15.6 °C. Calculated results have shown that the model is not sensitive to the variation of water density, but it is sensitive to the variation of specific heat and density of the rock especially in the fractured system. Also, the model is very sensitive to any variation on the water velocity in sinkhole points, and any variation in the depth of saturated zone. The influence of the variation of the groundwater slope along the study reach is small compared

<table>
<thead>
<tr>
<th>Well</th>
<th>Measured temperature</th>
<th>Calculated temperature</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berruet 1</td>
<td>15.1</td>
<td>15.7</td>
<td>3.9</td>
</tr>
<tr>
<td>Berruet 3</td>
<td>16.2</td>
<td>16</td>
<td>1.2</td>
</tr>
<tr>
<td>Berruet 4</td>
<td>13.3</td>
<td>15.1</td>
<td>13.5</td>
</tr>
<tr>
<td>Berruet 6</td>
<td>17.3</td>
<td>16</td>
<td>7.5</td>
</tr>
<tr>
<td>Moret</td>
<td>17.6</td>
<td>17.8</td>
<td>1.1</td>
</tr>
<tr>
<td>Boires 1</td>
<td>22.1</td>
<td>20.1</td>
<td>9</td>
</tr>
<tr>
<td>Ligne</td>
<td>14.4</td>
<td>15.1</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>12.9</td>
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<td>17.8</td>
</tr>
<tr>
<td>Moret 2</td>
<td>17.3</td>
<td>17.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Boires 2</td>
<td>18.8</td>
<td>18.5</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>20.5</td>
<td>20.5</td>
<td>0</td>
</tr>
<tr>
<td>Bouillon</td>
<td>15.6</td>
<td>15.1</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Table (1): Comparison between measured and calculated temperatures in wells.

In order to verify the accuracy of the present numerical model, a comparison between measured and calculated temperatures in wells is occurred. The best results are obtained when the water velocity in sinkhole points is 80 m/hr, the porosity is 10% in the fractured system and 80% in the conduit system and the depth of saturated zone is 25 m. Table (1) displays this comparison with the percentage error for each well. It can be clearly observed that the results of the model are very good compared with the measured temperature. The percentage error of the model ranges from zero to 17.8 percent.

Figure (12): Effect of the depth of saturated zone on the behavior of groundwater temperature distribution.

Figure (13): Effect of the density and specific heat of the rock on the behavior of groundwater temperature distribution.
with other parameters. The best results are occurred when the water velocity in sinkhole points is 80 m/hr, the porosity is 10% in the fractured system and 80% in the conduit system and the depth of saturated zone is 25 m. Finally, it was observed that the comparison between measured and calculated temperatures in wells is very good.

References


الملخص العربي

نماذج الحرارة في طبقة المياه الجوفية الكارستية لمدينة أورليانز (فرنسا)

الخلاصة

في الأوساط الكارستية، درجة الحرارة تلعب دور مهم خصوصاً لتحديد التغير في هذه الأوساط. هدف هذا العمل هو تطوير نموذج رياضي ثنائي الأبعاد لاستكشاف الحرارة في الكارست. معدالة نايفير ستوكس (Navier Stokes) معادلة انتقال الحرارة، استخدمت لقياس التغير في درجات الحرارة في الطبقات السطحية. النموذج الرياضي تم تطبيقه في النظام الكارستي في مدينة أورليانز (فرنسا). النتائج تظهر أن النموذج يمكن استخدامه لقياس التغير الحراري في كافة الصخور الكارستية خصوصاً في الوسط المتشقق. النموذج هو نموذج جيد جداً لقياس التغير في درجات الحرارة. النموذج يمكن استخدامه لقياس التغير الحراري في الأوساط الكارستيةUTC.